# Variation of Isotope Coefficient with Number of ${\rm CuO}_2$ Layers in High TC Superconductors

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### Abstract

In this paper, an effort has been made to study the isotope coefficient in high  $T_c$  Superconductors by using the variation of isotopic coefficient with the number of  $CuO_2$  layers and the variation of isotope coefficients on transition temperature  $T_c$ . The Hamiltonian for  $CuO_2$  layers using BCS type model and extra term of interlayer interaction between  $CuO_2$  layers has been considered. Expressions for isotope effect ( $\alpha$ ) and transition temperature ( $T_c$ ) are obtained and numerically calculated for experimental values by using Green's function technique.

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**Keywords:** Isotope effect;  $CuO_2$  layers; Isotope coefficient; High  $T_c$  superconductors

# Introduction

After the discovery of high T<sub>c</sub> cuprate superconductors in 1986 [1] at 35 K, the field is buzzing with research activities. Efforts to increase the transition temperature are currently going on. Till now the highest reached T<sub>c</sub> under pressure is 164 K in HgBa<sub>2</sub>Ca<sub>2</sub>Cu<sub>3</sub>O<sub>8+8</sub> [2]. These cuprates have unconventional properties both in normal and superconducting state [3,4]. Till date there is no consensus on the origin of pairing mechanism. It is now widely suggested that spin fluctuation driven pairing mechanism provides a good agreement between theory and experiments [5]. However, there are several experimental observations which clearly indicate that this purely electronic picture is incomplete and lattice effects have to be taken into account [6-8]. Isotope shift of T<sub>c</sub> is regarded as the defining signature of superconducting pairing resulting from phonons. In high T<sub>c</sub> superconductors, a small isotope effect is found [9-11]. Suppression of this effect is not explained within conventional BCS theory which predicts that the critical temperature T<sub>c</sub> and isotope mass M are related by  $T_{c} \propto M^{-\alpha}$  where  $\alpha$ =0.5 for all elements. To explain the small isotope effect in high T<sub>c</sub> compounds, many mechanisms including resonance valence bond [12], excitons [13], Plasmon's [14] and antiferromagnetic spin fluctuation mediated pairing are proposed [15]. Daemen and Overhauser [16] found that the existence of a short-range attraction in addition to the conventional phonon pairing interaction suppresses the isotope effect significantly at high temperatures. Here we present the variation of isotope coefficients with number of CuO<sub>2</sub> layers and the variation of isotope coefficients on transition temperature  $T_{\rm c}$  [17-22]. As the trilayer material has highest  $(T_c)$  in these cuprates but it has very small isotope coefficient ( $\alpha$ ). Clearly the argument that a mechanism other than an electron phonon interaction dominates the superconductivity based only on a small  $(\alpha)$  in a cuprate with relatively high  $(T_c)$  is inappropriate. The observed  $CuO_2$  layer dependence of the isotope effect indicates that the interlayer coupling between the adjacent CuO, planes is necessary for superconductivity in layered cuprates [23- $\overline{26}$ ]. For monolayer materials having lower (T<sub>c</sub>) the interlayer coupling plays a less important role and  $(T_c)$  can be mainly controlled by the phonon coupling yielding a larger size of isotope coefficient with increasing the number of CuO<sub>2</sub> layers in a unit cell, the interlayer coupling begins to play an important role in enhancing  $(T_c)$ and the isotope effect is expected to be small.

#### Formulation

The model Hamiltonian for our system can be described as

$$\begin{split} H &= \Pi_{\text{int}ra} + \Pi_{\text{int}er} \\ H_{\text{int}ra} &= \sum_{km\sigma} E_k a^+_{mk\sigma} a_{mk\sigma} - V \sum_{mkk^{\cdot}} a^+_{mk\uparrow} a^+_{m-k\downarrow} a_{m-k^{\cdot}\downarrow} a_{mk^{\cdot}\uparrow} \\ H_{\text{int}er} &= -W \sum_{mnkk^{\cdot}} a^+_{mk\uparrow} a_{n-k^{\cdot}\downarrow} a_{mk^{\cdot}\uparrow} \\ H &= \sum_{km\sigma} E_k a^+_{mk\sigma} a_{mk\sigma} - V \sum_{mkk^{\cdot}} a^+_{mk\downarrow} a^+_{m-k\downarrow} a_{mk^{\cdot}\uparrow} - W \sum_{mnkk^{\cdot}} a^+_{n-k\downarrow} a^+_{n-k\downarrow} a_{mk^{\cdot}\uparrow} (1) \end{split}$$

Where  $a_{mk\sigma}^+$ ,  $a_{mk\sigma}$  denote the fermions creation and annihilation operator respectively, k is the wave vector and  $\sigma$  is spin index for fermions.

In our present analysis we use a Green's function, defining as

$$G_{mnkk^{+}}^{\uparrow\uparrow} = \left\langle \left\langle a_{mk\uparrow}, a_{nk^{+}\uparrow}^{+} \right\rangle \right\rangle \tag{2}$$

Equation of motion is written as

$$\omega G_{mnkk^{+}}^{\uparrow\uparrow} = \frac{1}{2\pi} + \left\langle \left\langle \left[ a_{mk\uparrow}, H \right], a_{nk^{+}\uparrow}^{+} \right\rangle \right\rangle$$

Evaluating the commutator  $\begin{bmatrix} a_{m-k\downarrow}^+, H \end{bmatrix}$  using the Hamiltonian (1) we get

$$\left[a_{mk\uparrow},H\right] = E_{K}a_{mk\uparrow} - \sum_{k} (V)a_{m-k\downarrow}^{+}a_{m-k\downarrow}a_{mk\uparrow} - \sum_{nk} (W)a_{n-k\downarrow}^{+}a_{n-k\downarrow}a_{mk\uparrow}\right]$$

And writing the equation of motion as

$$\omega G_{mnkk^{+}}^{\uparrow\uparrow} = \frac{1}{2\pi} + \left\langle \left\langle \left[ a_{mk\uparrow}, H \right], a_{nk\uparrow}^{+} \right\rangle \right\rangle$$
(3)

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Putting the value of commutator  $\begin{bmatrix} a_{_{mk\uparrow}}, H \end{bmatrix}$  in equation (3) we get 

$$\omega G_{nmkk}^{\uparrow\uparrow} = \frac{1}{2\pi} + \left\langle \left\langle E_{k} a_{mk\uparrow}, a_{nk}^{+} \right\rangle \right\rangle - \sum_{k} V \left\langle \left\langle a_{m-k\downarrow}^{+} a_{m-k\downarrow} a_{mk\uparrow}, a_{nk\uparrow}^{+} \right\rangle \right\rangle - \sum_{k} W \left\langle \left\langle a_{n-k\downarrow}^{+} a_{n-k\downarrow} a_{mk\uparrow}, a_{nk\uparrow}^{+} \right\rangle \right\rangle$$

$$Or \left(\omega - E_{k}\right) G_{mnkk}^{\uparrow\uparrow} = \frac{1}{2\pi} - \left[ \Delta v + \sum_{n} \Delta W \right] G_{mn-kk\uparrow}^{\uparrow\uparrow} \qquad (42)$$

Now we introduce the order parameter  $\Delta$  such as

$$\Delta_{rr} = \sum_{k} V_{rr}(kq) \left\langle C_{rk\uparrow}^{+} C_{r-k\downarrow}^{+} \right\rangle, \qquad \Delta_{rj} = \sum_{k} V_{rj}(kq) \left\langle C_{jk\uparrow}^{+} C_{j-k\downarrow}^{+} \right\rangle$$

Where  $G_{mn-kk^{+}}^{\downarrow\uparrow}$  is another Green's function which may be written as  $G_{mn-kk^{+}}^{\downarrow\uparrow} = \left\langle \left\langle a_{m-k\downarrow}^{+}, a_{nk^{+}\uparrow}^{+} \right\rangle \right\rangle$  (5) This Green's function may also be written in terms of equation of

motion as

$$\omega G_{mn-kk}^{\downarrow\uparrow} = \left\langle \left\langle \left[ a_{m-k\downarrow}^{+}, H \right]; a_{nk\uparrow}^{+} \right\rangle \right\rangle$$
(6)

Evaluating the commutator  $[a_{m-k\downarrow}^+, H]$  using the Hamiltonian  $a_{m-k\downarrow}^{+}, H = \left[a_{m-k\downarrow}^{+}, \sum_{km\sigma} E_{k} a_{mk\sigma}^{+} a_{mk\sigma} - V \sum_{mkk} a_{mk\uparrow}^{+} a_{m-k\downarrow}^{+} a_{m-k\downarrow} a_{mk\uparrow} - W \sum_{mnkk} a_{mk\uparrow}^{+} a_{n-k\downarrow}^{+} a_{n-k\downarrow} a_{mk\uparrow} \right]$ Putting the commutator  $E_{(-k)} = E_{k}$  from (7) in equation (6) we get  $\omega G_{_{mn-kk}}^{\downarrow\uparrow} = -E_k \left\langle \left\langle a_{_{mk\downarrow}}^+, a_{_{mk}^+\uparrow}^+ \right\rangle \right\rangle - V \sum_k \left\langle \left\langle a_{_{mk\uparrow}}^+, a_{_{m-k+\downarrow}}^+ \right\rangle \right\rangle \left\langle \left\langle a_{_{mk\uparrow}}, a_{_{mk\uparrow}\uparrow}^+ \right\rangle \right\rangle - W \sum_{_{mk}^+} \left\langle \left\langle a_{_{mk\uparrow}}^+, a_{_{m-k+\downarrow}}^+ \right\rangle \right\rangle \left\langle \left\langle a_{_{mk\uparrow}}, a_{_{mk\uparrow}\uparrow}^+ \right\rangle \right\rangle - W \sum_{_{mk}^+} \left\langle \left\langle a_{_{mk\uparrow}\uparrow}^+, a_{_{mk+\downarrow}\uparrow}^+ \right\rangle \right\rangle = 0$ 

But from the law of conservation of energy

$$E_{(-k)} = E_{k}$$

$$(\omega + E_{k})G_{mn-kk}^{\downarrow\uparrow} = -\left(\Delta V + \sum_{\substack{n \\ m \neq k}} \Delta W\right)G_{mnkk}^{\uparrow\uparrow}$$
(8)

In equation (4) we finally obtained the equation

$$\left(\omega - E_{k}\right)G_{mnkk}^{\uparrow\uparrow} = \frac{1}{2\pi} - \left[\Delta V + \sum_{mn} \Delta_{n}W\right]G_{mn-kk}^{\downarrow\uparrow}$$
(9)

Multiply  $(\omega + E_k)$  in equation (9) and putting the value of  $(\omega - E_k) G_{mnkk}^{\uparrow\uparrow}$ from equation (8) then we get the Green's function  $G_{_{\it mnkk^{\circ}}}^{_{\uparrow\uparrow}}$  as

$$\left(\omega^{2} - E_{k}^{2}\right)G_{mode}^{\text{II}} = \frac{1}{2\pi}\left(\omega + E_{k}\right) - \left(\Delta V + \sum_{n} \Delta W\right)\left(\omega + E_{k}\right)G_{mode}^{\text{II}}$$
(10)

Multiplying by  $(\omega + E_k)$  in the equation (8) and putting the value of  $(\omega - E_k) G_{mnkk}^{\uparrow\uparrow}$  from equation (9) we get the Green's function as

$$G_{mnkk^{*}}^{\uparrow\uparrow} = \frac{\omega + E_{k}}{2\pi \left[ \left( \omega^{2} - E_{k}^{2} \right) + \left( \Delta V + \sum_{n} \Delta_{n} W \right) \left( \Delta V + \sum_{n} \Delta_{n} W \right) \right]}$$
$$\left( \omega^{2} - E_{k}^{2} \right) G_{mnkk^{*}}^{\downarrow\uparrow} = - \left( \Delta V + \sum_{m \neq n} \Delta_{n} W \right) \left( \omega - E_{k} \right) G_{mnkk^{*}}^{\uparrow\uparrow}$$

We know from equation (9)

$$G_{mn-kk^{-}}^{\downarrow\uparrow} = -\frac{\left(\Delta V + \sum_{n} \Delta W\right)}{2\pi \left(\omega^{2} - E_{k}^{2} - \left(\Delta V + \sum_{n} \Delta_{n} W\right)^{2}\right)}$$
(11)

Using the Green's function, we can obtain the expression for order

parameter  $\Delta_{_{\!\! mn}}$  and correlation parameter  $\gamma$  the order parameter  $\Delta_{_{\!\! mn}}$ may be written as

$$\Delta_{mn} = \sum_{k} V \left\langle a^+_{mk^+} a^+_{m-k^+} \right\rangle$$
(12)

Correlation function  $\langle a^+_{mk^+\uparrow}a^+_{m-k^+\downarrow}\rangle$  is related to Green's function  $G^{\downarrow\uparrow}_{mn-kk^+}$  as

$$\left\langle a_{mk^{+}\uparrow}^{+}a_{m-k^{+}\downarrow}^{+}\right\rangle = -\frac{1}{i}\int_{-\infty}^{\infty}\frac{G_{mn-kk^{+}}^{\downarrow\uparrow}\left(\omega+i\varepsilon\right) - G_{mn-kk^{-}}^{\uparrow\uparrow}}{e^{\gamma_{KT}^{\prime}} - \eta}d\omega$$
(13)

Where  $\eta$ =-1 for fermions, K=Boltzmann constant and T= Temperature

Green's function  $G_{mn-kk}^{\downarrow\uparrow}(\omega+i\varepsilon)$  and  $G_{mn-kk}^{\downarrow\uparrow}(\omega-i\varepsilon)$  may be expressed as

$$G_{m-k}^{ii}(\omega+i\varepsilon) = \frac{-\left(\Delta V + \sum_{s} \Delta_{s} W\right)}{2\pi \left[\left(\omega+i\varepsilon\right)^{2} - E_{s}^{2} - \left(\Delta V + \sum_{s} \Delta_{s} W\right)^{2}\right]}$$
(14)  
$$\therefore \xi_{q}^{2} = E_{k}^{2} + \left(\Delta V + \sum_{n} \Delta_{n} W\right)^{2}$$
$$G_{m-k}^{i\uparrow}(\omega+i\varepsilon) = -\frac{\left(\Delta V + \sum_{n} \Delta_{n} W\right)}{4\pi \xi_{q}} \left\{\frac{(\omega-\xi_{q})}{(\omega-\xi_{q})^{2} - \varepsilon^{2}} - i\pi \delta(\omega-\xi_{q}) - \frac{(\omega+\xi_{q})}{(\omega-\xi_{q})^{2} - \varepsilon^{2}} + i\pi \delta(\omega+\xi_{q})\right\}$$
(15)

$$G_{mn-kk}^{\downarrow\uparrow}(\omega-i\varepsilon) = \frac{-\left(\Delta V + \sum_{n} \Delta_{n}W\right)}{2\pi \left[\left(\omega-i\varepsilon\right)^{2} - E_{k}^{2} - \left(\Delta V + \sum_{n} \Delta_{n}W\right)^{2}\right]}$$
(16)

$$\therefore \xi_{q}^{2} = E_{k}^{2} + \left(\Delta V + \sum_{n} \Delta_{n} W\right)^{2}$$

$$G_{mn-kk}^{\uparrow\uparrow}(\omega - i\varepsilon) = \frac{-\left(\Delta V + \sum_{n} \Delta_{n} W\right)}{2\pi \left[\left(\omega - i\varepsilon\right)^{2} - \xi_{q}^{2}\right]}$$

$$G_{mn-kk}^{\downarrow\uparrow}(\omega - i\varepsilon) = -\frac{\left(\Delta V + \sum_{n} \Delta_{n} W\right)}{4\pi \xi_{q}} \left\{\frac{\left(\omega - \xi_{q}\right)}{\left(\omega - \xi_{q}\right)^{2} + \varepsilon^{2}} + i\pi\delta\left(\omega - \xi_{q}\right) - \frac{\left(\omega - \xi_{q}\right)}{\left(\omega - \xi_{q}\right)^{2} + \varepsilon^{2}} + i\pi\delta\left(\omega - \xi_{q}\right)\right\} (17)$$

Substitute both the Green's function  $G_{mn-kk^{-}}^{\downarrow\uparrow}(\omega+i\varepsilon)$  and  $G_{mn-kk^{-}}^{\downarrow\uparrow}(\omega-i\varepsilon)$  from equation (13) and then after

solving we get correlation function.  

$$G_{mn-kk}^{\downarrow\uparrow}(\omega+i\varepsilon) - G_{mn-kk}^{\downarrow\uparrow}(\omega-i\varepsilon) = -\frac{\left(\Delta V + \sum_{n} \Delta_{n} W\right)}{4\pi\xi_{q}} \left(2i\pi\delta\left(\omega-\xi_{q}\right) - 2i\pi\delta\left(\omega+\xi_{q}\right)\right) \quad (18)$$

$$\therefore \left\langle a_{mk\uparrow}^{+}a_{m-k\downarrow}^{+}\right\rangle = -\frac{1}{i} \int_{-\infty}^{\infty} \frac{G_{mn-kk\uparrow}^{\downarrow\uparrow}(\omega+i\varepsilon) - G_{mn-kk\uparrow}^{\downarrow\uparrow}}{e^{\% KT} - \eta} d\omega$$
Where  $\eta = -1$ 

$$\left\langle a_{mk\uparrow}^{+}a_{m-k\downarrow}^{+}\right\rangle = -\frac{1}{i} \int_{-\infty}^{\infty} -\frac{\left(\Delta V + \sum_{n} \Delta_{n} W\right)}{4\pi\xi_{q}} \left(\frac{2i\pi\delta\left(\omega-\xi_{q}\right) - 2i\pi\delta\left(\omega+\xi_{q}\right)}{e^{\% KT} - \eta}\right) d\omega$$

$$\left\langle a_{mk\uparrow}^{+}a_{m-k\downarrow}^{+}\right\rangle = -\frac{1}{i} \int_{-\infty}^{\infty} -\frac{\left(\Delta V + \sum_{n} \Delta_{n} W\right)}{4\pi\xi_{q}} \left(\frac{2i\pi\delta\left(\omega-\xi_{q}\right) - 2i\pi\delta\left(\omega+\xi_{q}\right)}{e^{\% KT} - \eta}\right) d\omega$$

$$\left\langle a_{mk\uparrow}^{+}a_{m-k\downarrow}^{+}\right\rangle = -\frac{1}{i} \int_{-\infty}^{\infty} -\frac{\left(\Delta V + \sum_{n} \Delta_{n} W\right)}{4\pi\xi_{q}} \left(\frac{2i\pi\delta\left(\omega-\xi_{q}\right) - 2i\pi\delta\left(\omega+\xi_{q}\right)}{e^{\% KT} - \eta}\right) d\omega$$

$$\left\{ a_{mk\uparrow}^{+}a_{m-k\downarrow}^{+}\right\} = -\frac{1}{i} \int_{-\infty}^{\infty} -\frac{\left(\Delta V + \sum_{n} \Delta_{n} W\right)}{4\pi\xi_{q}} \left(\frac{2i\pi\delta\left(\omega-\xi_{q}\right) - 2i\pi\delta\left(\omega+\xi_{q}\right)}{e^{\% KT} - \eta}\right) d\omega$$

$$\left\{ a_{mk\uparrow}^{+}a_{m-k\downarrow}^{+}\right\} = -\frac{1}{i} \int_{-\infty}^{\infty} -\frac{\left(\Delta V + \sum_{n} \Delta_{n} W\right)}{4\pi\xi_{q}} \left(\frac{2i\pi\delta\left(\omega-\xi_{q}\right) - 2i\pi\delta\left(\omega+\xi_{q}\right)}{e^{\% KT} - \eta}\right) d\omega$$

$$\left\{ a_{mk\uparrow}^{+}a_{m-k\downarrow}^{+}\right\} = -\frac{1}{i} \int_{-\infty}^{\infty} -\frac{\left(\Delta V + \sum_{n} \Delta_{n} W\right)}{4\pi\xi_{q}} \left(\frac{2i\pi\delta\left(\omega-\xi_{q}\right) - 2i\pi\delta\left(\omega+\xi_{q}\right)}{e^{\% KT} - \eta}\right) d\omega$$

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$$\left\langle a_{mk^{+}\uparrow}^{+}a_{m-k^{-}\downarrow}^{+}\right\rangle = \frac{\left(\Delta V + \sum_{n}\Delta_{n}W\right)}{2\xi_{q}} \left[\frac{1}{e^{x}+1} - \frac{1}{e^{-x}+1}\right]$$
(19)

$$\therefore \frac{1}{e^{x} + 1} - \frac{1}{e^{-x} + 1} = \frac{e^{-\frac{x}{2}} + e^{\frac{x}{2}}}{e^{-\frac{x}{2}} + e^{\frac{x}{2}}} = \tanh \frac{x}{2} = \tanh \frac{\xi_{q}}{2kT} \quad \text{Using in}$$

(19) we get

$$\left\langle a_{sst}^{*} \uparrow a_{s-t}^{*} \right\rangle = \frac{\left(\Delta V + \sum_{s} \Delta_{s} W\right)}{\xi_{q}} \left( \tanh \frac{\xi_{q}}{2kT} \right) = \frac{\Delta V + \sum_{s} \Delta_{s} W}{\sqrt{E_{q}^{2} + \left(\Delta V + \sum_{s} \Delta_{s} W\right)^{2}}} \tanh \sqrt{E_{q}^{2} + \left(\Delta V + \sum_{s} \Delta_{s} W\right)^{2}}$$
(20)

Then we can obtained the expression of or dr parameter  $\Delta_{ir}$  by substituting correlation function in equation (12)

$$\Delta_{ir} = \sum_{k} V_{ir} \left( k_{q}^{*} \right) \left\langle c_{ik^{+}\uparrow}^{+} c_{i-k^{+}\downarrow}^{+} \right\rangle = \sum_{k} V_{ir} \left( k_{q}^{*} \right) \left\langle a_{mk^{+}\uparrow}^{+} a_{m-k^{+}\downarrow}^{+} \right\rangle$$
$$\Delta_{mn} = \sum_{k} V_{ir} \left( k_{q}^{*} \right) \frac{\Delta V + \sum_{n} \Delta_{n} W}{\sqrt{E_{q}^{2} + \left( \Delta V + \sum_{n} \Delta_{n} W \right)^{2}}} \tanh \frac{\sqrt{E_{q}^{2} + \left( \Delta V + \sum_{n} \Delta_{n} W \right)^{2}}}{2kT}$$

Converting summation over K into integration with cut-off energy  $\hbar \omega_p$  from the Fermi level we get

$$\Delta_{ss} = N_{0} \int_{0}^{had} V_{s}\left(k_{s}\right) \frac{\left(\Delta V + \sum_{s} \Delta_{s}W\right)}{\sqrt{E_{s}^{2} + \left(\Delta V + \sum_{s} \Delta_{s}W\right)^{2}}} \tanh \frac{\sqrt{E_{s}^{2} + \left(\Delta V + \sum_{s} \Delta_{s}W\right)^{2}}}{2kT} dE_{s} \quad (21)$$

$$\cdot \frac{1}{N_{0}V} = \int_{0}^{had} \frac{1}{\sqrt{E_{q}^{2} + \left(\Delta V + \sum_{n} \Delta_{n}W\right)^{2}}} \tanh \frac{\sqrt{E_{q}^{2} + \left(\Delta V + \sum_{n} \Delta_{n}W\right)^{2}}}{2k_{B}T} dE_{q} \quad (22)$$

At 
$$T = T_C, \Delta = 0$$
  

$$\frac{1}{N_0 V} = \frac{1}{2K_B T_C} \int_0^{h\omega_D} \frac{1}{\left(\frac{E_q}{2K_B T_C}\right)} \tanh \frac{E_q}{2k_B T_C} dE_q \qquad (23)$$

$$\therefore \frac{1}{N_0 V} = \frac{1}{2K_B T_C} \int_0^{\frac{h \alpha_D}{2K_B T_C}} \frac{1}{x} \tanh x (2k_B T_C) dx$$

Or  $\frac{\hbar\omega_D}{2K_B T_C} e^{0.8185} = \exp^{\frac{1}{N_0 V}}$  $\therefore T_C = (1.13072) \frac{\hbar\omega_D}{2K_B} \exp^{\frac{1}{N_0 V}}$ (24)

The Isotope effect coefficient is

$$\alpha = -\frac{d\ln T_c}{d\ln M} \tag{25}$$

Using equation (29) and (30) we get

$$\alpha = \frac{d}{d \ln M} \left[ \log(1.13072) \frac{\hbar \omega_D}{K_B} + \log e^{-\frac{1}{N_0 V}} \right]$$
(26)

Put 
$$T_C \propto M^{-2}$$
,  $or T_C = KM^{-2}$   
 $\therefore \alpha = -\frac{d}{4d \log V} [2 \log V] - \frac{d}{4d (\log V)} \left(\frac{1}{N_0 V}\right)$ 
(27)

Differentiating we get

$$\frac{1}{V_0 V} = \left(2(1+2\alpha)\right) \tag{28}$$

Now from equation (24) we get

$$T_{C} = 1.13072 \frac{\hbar\omega_{D}}{K_{B}} \exp\left(-2\left(2\alpha + 1\right)\right)$$
<sup>(29)</sup>

By solving equation (33), we get  $\alpha$  as

$$\alpha = \frac{1}{4N_0 V \left(1 + \frac{D}{N_0}\right)} - \frac{1}{2}$$
(30)

The values of  $\alpha$  and T<sub>c</sub> are calculated from equation (29) and (30) respectively for various values of different *CuO*<sub>2</sub> layers (Table 1).

# **Results and Discussion**

The numerical analysis of equation (29) and (30) have been calculated for different  $CuO_2$  layers taking  $\frac{D}{N_0} = 0.2(forn = 1)$  and  $\frac{D}{N_0} = 0.6(forn = 3)$ . Figure 1a is plotted between isotope coefficient ( $\alpha$ ) and number of CuO<sub>2</sub> layers (n) per unit cell. Figure shows that as increasing number of  $CuO_2$  layers per unit cell, isotope coefficient ( $\alpha$ ) linearly decreases. For n=1, the value of ( $\alpha$ ) is 0.34 and it decreases continuously with number of  $CuO_2$  layers. At n=3, isotope coefficient ( $\alpha$ ) is 0.1317 which is very low. This systematic reduction of isotope coefficient with increasing CuO, layers indicates that isotope effect can be negligible for multilayer cuprates. This result is good agreement with experimental result made by Chen et al. Figure 1b is plotted between transition temperature  $(T_c)$  and isotope coefficient. The curve shows that as increasing ( $\alpha$ ), the transition temperature exponentially decreases so for lower isotope coefficient  $T_{c}$  will be maximum and  $T_{c}$  decreases for higher values of (a). So we can say that as increasing number of  $CuO_2$  layers, (a) decreases and the transition temperature increases. This result also supported by Chakravarty et al.

# Conclusion

First of all Hamiltonian for CuO2 layers using BCS type model



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|  | S No | α    | T <sub>c</sub> |
|--|------|------|----------------|
|  | 1    | 0.05 | 37.586         |
|  | 2    | 0.1  | 30.772         |
|  | 3    | 0.2  | 20.627         |
|  | 4    | 0.3  | 13.827         |
|  | 5    | 0.4  | 9.268          |
|  | 6    | 0.5  | 6.212          |
|  | 7    | 0.6  | 4.164          |
|  | 8    | 0.7  | 2.791          |

**Table 1:** Table for values of ( $\alpha$ ) and (T<sub>c</sub>).

and extra term of interlayer interaction between CuO<sub>2</sub> layers has been considered, using this equation expression for isotope effect ( $\alpha$ ) and transition temperature (T<sub>c</sub>). Expression obtained is numerically solved using Green's function technique and then the value of isotope effect ( $\alpha$ ) and transition temperature (T<sub>c</sub>) is calculated. The graph is plotted between isotopic coefficient and number of CuO, layers. The second graph is plotted between isotope effect ( $\alpha$ ) and transition temperature  $(T_c)$ . Then conclusion is drawn comparing with available experimental result. The tri layer material has highest  $(T_c)$  in these cuprates but it has very small isotope coefficient ( $\alpha$ ). Clearly the argument that a mechanism other than an electron-phonon interaction dominates the superconductivity based only on a small  $(\alpha)$  in a cuprate with relatively high  $(T_c)$  is inappropriate. The observed  $CuO_2$  layer dependence of the isotope effect indicates that the interlayer coupling between the adjacent CuO<sub>2</sub> planes is necessary for superconductivity in layered cuprates. For monolayer materials having lower  $(T_c)$  the interlayer coupling plays a less important role and  $(T_c)$  can be mainly controlled by the phonon coupling yielding a larger size of isotope coefficient with increasing the number of CuO, layers in a unit cell, the interlayer coupling begins to play an important role in enhancing  $(T_c)$  and the isotope effect is expected to be small.

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