A Theoretical Model of Corruption Using Modified Lotka Volterra Model: A Perspective of Interactions between Staff and Students

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Authors' contributions

This work was carried out in collaboration among all authors. Author MK designed the study, managed the literature searches and performed the statistical analysis. In addition author MK managed the analyses of the study, wrote the protocol and the first draft of the manuscript. Authors CGN and SK read and approved the final manuscript.

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Abstract

Corruption is the misuse of power or resources for private gain. This undermines economic development, political stability, and government legitimacy, the society fabric, allocation of resources to sectors crucial for development, and encourages and perpetuates other illegal opportunities. Despite Mathematical modeling being a powerful tool in describing real life phenomena it still remains unexploited in the fight of corruption menace. This study uses Lotka Volterra, predator-prey equations to develop a model to describe corruption in institutions of higher learning, use the developed model to determine its equilibria, determine the condition for stability of the equilibria and finally carry out the simulation. The corrupt students and staff act as predators while their non-corrupt counterparts act as prey in the paper. Theory of ordinary differential equations was used to determine steady states and their stability. Mathematica was

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used for algebraic analysis and Matlab was used for numerical analysis and simulation. Analytical result suggested multiple steady state however numerical result confirmed that the model has four steady states. Numerical bifurcation analysis suggests the possibility of backward of corrupt staff when β_2 is about 39. Numerical simulation points to an increasing trend on corrupt staff and decrease trend on corrupt student. This study concludes that more focus should be put to staff than students in curbing the spread of corruption. Future study should strive to fit this model in real data

1 Introduction

The research study defines corruption as a misuse of power or resources for private gain [1]. Corruption is not a unique problem to Kenya; it has always been a major challenge to mankind. Information available indicates corruption in Kenya can be traced since independence [2].

Corruption manifests itself in different forms including bribery, theft and fraud, and institutional. The following are the various types of corruption: grand corruption, political corruption, corporate corruption, administrative corruption, petty corruption, and systemic corruption. Although the causes of corruption are varied, they are linked to official's accountability in executing, such discretion due to ambiguous laws and regulations, poor enforcement of the law, and order among others. The overall culture of Governance is also known to play an important role in corruption [2].

Corruption may undermine economic development, political stability and government legitimacy, the society fabric, allocation of resources to sectors crucial for development and encourages and perpetuates other illegal opportunities [2].

The Kenya 2010 constitution recognized the burden of corruption in Kenya, and created a room for Parliament to enact legislation to establish an independent ethics and anti-corruption commission [3]. Several institutional frameworks have been created to mitigate corruption in Kenya which includes the following among others; EACC, Ombudsman, and PPOA [2]. According to the research study [4]. The research carried out in 2015 found that the majority of Kenyans acknowledge that corruption has increased over 2014 whereby nearly a third have had to pay a bribe [5].

The 2010 constitution of Kenya defines a state office as any State officer means a person holding a state office [3].

A public officer means any state officer or any person, who holds a public office. Public office means an office in the national government, a county government or the public service, if the remuneration and the benefits of the officer are payable directly from the consolidated fund or directly out of the money provided by parliament. Public service means the collectivity of all individuals, other than state officers, performing a function within a state organ [3].

Evidence suggests that Kenyans perceive many of the key sectors and institutions to be corrupt. The order of most corrupt sectors in 2011 is as follows; police, political parties, parliament among many others [2].

The classical Lotka - Volterra equations were discovered independently by Vito Volterra and Alfred Lotka in 1925-26. If Y(t) is the prey population and D(t) that of the predator at time t then Volterra's model is

$$\frac{dY}{dt} = Y(a - bD)$$

$$\frac{dD}{dt} = D(cY - d)$$

where a, b, c, and d are positive constants [6].

This research is based on the consideration that, no developed model describes the dynamics of corruption in institutions of higher learning as students interact with academic staff. Deterministic models are recommended when the population size is large or as "introductory models" when studying new phenomena. In compartmental models, the target population is divided into compartments or categories. The compartments often reflect the stages that individuals may progress through during an infection (in case of diseases). An individual can only belong to a category at any given time [7].

This research modified Lotka-Volterra model in modeling competitions of scarce resources. Scarce resources institution of higher includes among other few lecturer to students, finances etc. Just as in ecology inter and intra competitions, students and staff compete amongst themselves and across. Just like diseases corrupt thoughts, ideas, and attitudes are passed from corrupt individual(s) to susceptible individual(s) after effective contacts, "so is corrupt thoughts, ideas, and attitudes" passed from corrupt individuals to non-corrupt citizens. Susceptible(s) population in our model is assumed to be non-corrupt; staff, or students. Predators in our model are assumed to be staff or students with corrupt thoughts, ideas attitudes.

Schelling (1978) suggested that the expected profitability of engaging in corruption depends on its prevalence. The main result of the Schelling diagram was the existence of multiple equilibria and a tipping point. This paper puts Schelling's essentially static approach into an intertemporal setting and showed the existence of an unstable interior steady state [8].

A mathematical model for corruption as a disease in a population was subdivided into four (4) different classes according to their corruption status. The basic reproduction number was obtained as in epidemiological approach. The results revealed a globally asymptotically stable corruption-free equilibrium point whenever and a globally asymptotically stable endemic equilibrium point. Numerical simulations were carried out which validated the analytical results and further revealed that corruption can only be reduced to a bearable level but not eliminated [9].

This paper studied corruption as an epidemic phenomenon using the epidemic diffusion model of Kermack and Mc-Kendrick (1927) and sought to determine the dynamics of corruption and its effects on the composition of the population at a given time. The study determined a threshold of epidemiological corruption based on the approximation of the honest population [10].

This paper studied the relationship between corruption in public procurement and economic growth within the Solow framework in discrete time, whereas assuming that the public good is an input in the productive process and that the State fixes a monitoring level on corruption. This study demonstrated that no long-run equilibria with zero corruption exist and that periodic or aperiodic fluctuations in economic growth were likely to emerge. As a result, the economic system may be unpredictable or structurally unstable [11].

This study used a modified predator–prey model with transmissible disease. In this case, the predators were the police and the prey the gang members. The system exhibited five steady states—four of which involve no core gang members and one in which all the populations cohabit. Thresholds were identified which determined when the predator and prey populations persist and when the disease remains endemic. It was found that where the spread of disease among the police officers is greater than the death of the police officers, the diseased predator population survives [12].

This paper extended the Solow growth model to include corruption as a determinant of the multifactor productivity using a Cobb-Douglas production function framework. Corruption was incorporated as a determinant of government expenditure, investment, and foreign aid. It was found that output and growth are influenced by the level of corruption which was validated using Lebanon based time series data [13].

This study described prevention and disengagement strategies mathematically using an epidemiological compartmental model. The prevention and disentanglement strategies were modeled using model

parameters. The population vulnerable to corrupt ideologies was divided into three compartments: Susceptible class, Corrupted class, and corrupt political/ sympathizer class. Results were analyzed using basic reproduction numbers as in the epidemiological approach [14].

This study limited its consideration to comparing the effectiveness of corruption suppression in different levels of hierarchy (from the "costs-vs-gains" viewpoint) and found that the most profitable tactic consists in suppressing the lowest bureaucrats. A prospect of integration of the proposed system-social and game theory approaches into the general model was revealed [15].

This study varied four organizational culture-related parameters, i.e., organization structure, location of bad apples, employees' propensity to become corrupted (''corruption probability''), and number of whistle-blowers. Findings of simulation studies suggested that in organizations with flatter structures, corruption invades the organization at a lower threshold value of corruption probability compared to those with taller structures. Nevertheless, the final proportion of corrupted Individuals are higher in the latter as compared to the former [16].

This paper studied the effect of corruption on the Development of the society or country of the world. This study demonstrated how Mathematical modeling can be used to provide a solution to corruption menace [17].

This paper discussed that teaching mathematics is a valuable subject in inculcating good and acceptable behaviors and values among different categories of students in Nigeria. Some vital concepts in mathematics such as formulas, logic, and mathematical games can inculcate good behavior [18].

For a staff to be corrupt two factors play an important role; institutional checks and controls and molarity level. The goal of this research study is to combine the two factors to one parameter and evaluate how they contribute to the dynamics of corruption. Morality level, peer pressure among others play an important role for ordinary students to be corrupt, our research study will determine their effects as a single parameter in the dynamics of corruption. The effect of various intervention strategies such as the judicial process and call for individuals to abandon corrupt practices will be determined.

The adopted mathematical model will give insight into observed corrupt patterns, underlying mechanism which influences the recruitment of corrupt individuals, effects of the voluntary quitting. Furthermore, models can predict future trends, may determine optimal control strategies, and can supplement the research in absence of phenomenological data.

2 Model Formulation

A modified Predator-prey model was formulated where the predators are assumed to be corrupt staff and students. Non-corrupt staff and students are assumed to be prey in this study. Although the University community is diverse, this study considered interactions between student and staff as the epicenter of corruption. A population based compartment model was formulated where: N(t) represented the total population of students and staff, S(t), and W(t) represented non corrupt staff and students respectively and finally ($Sp(t)$) and $W_p(t)$ represented corrupt staff and students respectively. The rate at which students join through admission and exit institutions after completion are π_1 and μ_1 respectively. The rate at which staff join through recruitment and exit institution are π_2 and μ_2 respectively. It is assumed new corrupt individuals are recruited when they come into contact with either corrupt staff or corrupt student(s) or both. However, students are assumed to be more susceptible to corruption than staff. The force of corruption is expressed as $\beta_2 \frac{(S_P + \eta_2 W_P)}{N}$ and $\beta_1 \frac{(S_P + \eta_1 W_P)}{N}$ for staff and students respectively. The parameters β_1 and β_2 represent corruption rates for staff and students respectively. The parameters θ_1 and θ_2 represent voluntary quitting rates from corruption practices by corrupt student(s) and corrupt staff respectively.
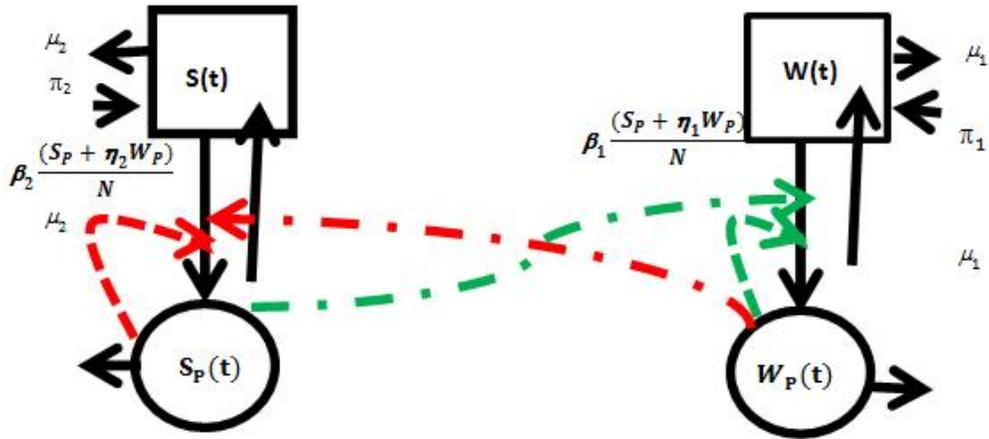
Diagram 1. The diagram below represent model flowchart

From the above flow chart we obtain the following ordinary differential systems of equation

$$\frac{dW}{dt} = \pi_1 W\left(1 - \frac{W}{K_1}\right) + \theta_1 W_P - \beta_1\left(\frac{S_P}{N} + \eta_1\frac{W_P}{N}\right)W - \mu_1 W \qquad (2.1),$$

$$\frac{dS}{dt} = \pi_2 S\left(1 - \frac{S}{K_2}\right) - \beta_2\left(\frac{S_P}{N} + \eta_2\frac{W_P}{N}\right)S - \mu_2 S \qquad (2.2),$$

$$\frac{dS_p}{dt} = \beta_2\left(\frac{S_P}{N} + \eta_2\frac{W_P}{N}\right)S - (\theta_2 + \mu_2)S_P \qquad (2.3),$$

$$\frac{dW_p}{dt} = \beta_1\left(\frac{S_P}{N} + \eta_1\frac{W_P}{N}\right)W - (\theta_1 + \mu_1)W_P \qquad (2.4),$$

$$N = S + W + S_p + W_p$$

Let τ be the proportion of students then $W + W_p = \tau N$ and $S + S_p = (1 - \tau)N$

$$\frac{dN}{dt} = \frac{dW}{dt} + \frac{dG}{dt} + \frac{dG_p}{dt} + \frac{dW_p}{dt} = \pi_1 W\left(1 - \frac{W}{K_1}\right) + \pi_2 S\left(1 - \frac{S}{K_2}\right) - \mu_1\tau N - \mu_2(1 - \tau)N$$

The initial conditions of the model are $W(0) \geq 0$; $S(0) \geq 0$; $S_p(0) \geq 0$; $W_p(0) \geq 0$

$$\frac{dN}{dt} = \omega - \mu_1\tau N - \mu_2(1 - \tau)N$$

3 Model Analysis

3.1 Boundednes

Theorem 1

$$\lim_{t\to\infty} N(t) = \frac{\omega}{[\mu_1\tau + \mu_2(1 - \tau)]}$$

Proof

Let N be the total population, then $\frac{dN}{dt} = \omega - \left[\mu_1 \tau + \mu_2(1-\tau)\right]N$

$$\frac{dN}{dt} + \left[\mu_1 \tau + \mu_2(1-\tau)\right]N = \omega$$

Applying integrating factor method,

$$N = \frac{\omega}{\left[\mu_1 \tau + \mu_2(1-\tau)\right]} + Ce^{-\left[\mu_1 \tau + \mu_2(1-\tau)\right]t}$$

3.2 Feasible region of the system

Let $s = \frac{S}{N}$, $s_p = \frac{S_p}{N}$, $w = \frac{W}{N}$, $w_p = \frac{W_p}{N}$ and t = t. Therefore $s + s_p = \tau \; and \; w + w_p = 1 - \tau$

The system of equations $(2.1) - (2.4)$, reduces to

$$\frac{ds_p}{dt} = \beta_2(s_p + \eta_2 w_p)(\tau - s_p) - (\theta_2 + \mu_2)s_p \tag{3.2.1},$$

$$\frac{dw_p}{dt} = \beta_1(s_p + \eta_1 w_p)(1 - \tau - w_p) - (\theta_1 + \mu_1)w_p \tag{3.2.2}$$

$$\frac{dW}{dt} = \pi_1 W\left(1 - \frac{W}{K_1}\right) + \theta_1 W_P - \beta_1(s_p + \eta_1 w_p)W - \mu_1 W \tag{3.2.3}$$

$$\frac{dS}{dt} = \pi_2 S\left(1 - \frac{S}{K_2}\right) - \beta_2(s_p + \eta_2 w_p)S - \mu_2 S \tag{3.2.4}$$

Consider equation (3.2.1), that is $\frac{ds_p}{dt} = \tau\beta_2(s_p + \eta_2 w_p) - (\beta_2(s_p + \eta_2 w_p)\theta_2 + \mu_2)s_p$. Clearly the term $\tau\beta_2(s_p + \eta_2 w_p) \geq 0$.

When positive term is dropped we obtain $\frac{ds_p}{dt} \geq -(\beta_2(s_p + \eta_2 w_p)\theta_2 + \mu_2)s_p$. On integration $s_p(t) \geq s_p(0)e^{-\int_0^t (\beta_2(s_p(s) + \eta_2 w_p(s))\theta_2 + \mu_2)ds} \geq 0$

Similarly $w_p(t) \geq w_p(0)e^{-\int_0^t \{\beta_1(s_p(s) + \eta_1 w_p(s))(\tau + w_p(s)) + (\theta_1 + \mu_1)w_p(s)\}ds} \geq 0$

3.3 Steady states

3.3.1 Steady states

If none of students or staff is corrupt then $E^0 = (w_p^0, s_p^0) = (0,0)$. If none of staff is corrupt then $s_p^1 = 0$ and steady state $E^1 = (w_p^1, s_p^1)$ is obtained by setting equations(3.2.1) and (3.2.1) to zero to obtain $\beta_1 \eta_1 w_p^1(1 - \tau - w_p^1) - (\theta_1 + \mu_1)w_p^1 = 0$. Note that $w_p^1 = 0$ corresponds to E^0. Therefore $E^1 = (w_p^1, s_p^1) = \left(-\frac{(-1+\tau)\beta_1\eta_1 + \theta_1 + \mu_1}{\beta_1\eta_1}, 0\right)$. If none of students is corrupt then $w_p^2 = 0$ and steady state $E^2 = (w_p^2, s_p^2)$ is obtained by setting equations(3.2.1) and (3.2.1) to zero to obtain $\beta_2 s_p^2(\tau - s_p^2) - (\theta_2 + \mu_2)s_p^2 = 0$. Note that $s_p^2 = 0$ correspond to E^0. Hence $E^2 = (w_p^2, s_p^2) = \left(0, -\frac{-\tau\beta_2 + \mu_2 + \theta_2}{\beta_2}\right)$. If both students and staff are

prone to corruption then steady state $E^3 = (w_p^3, s_p^3)$ is obtained by setting equations(3.2.1) and (3.2.1) to zero to obtain

$$s_{p1}^3 = \frac{\tau\beta_2 - w_p^3\beta_2\eta_2 - \theta_2 + \sqrt{4w_p^3\tau\beta_2^2\eta_2 + (\tau\beta_2 - w_p^3\beta_2\eta_2 - \theta_2 - \mu_2)^2} - \mu_2}{2\beta_2},$$

$$s_{p2}^3 = -\frac{-\tau\beta_2 + w_p^3\beta_2\eta_2 + \theta_2 + \sqrt{4w_p^3\tau\beta_2^2\eta_2 + (\tau\beta_2 - w\beta_2\eta_2 - \theta_2 - \mu_2)^2} + \mu_2}{2\beta_2}$$

$$w_{p1}^3 = -\frac{s_p^3\beta_1 - \beta_1\eta_1 + \tau\beta_1\eta_1 + \theta_1 - \sqrt{4\beta_1(s_p^3\beta_1 - s\tau\beta_1)\eta_1 + (-s_p^3\beta_1 + \beta_1\eta_1 - \tau\beta_1\eta_1 - \theta_1 - \mu_1)^2} + \mu_1}{2\beta_1\eta_1},$$

$$w_{p2}^3 = -\frac{s_p^3\beta_1 - \beta_1\eta_1 + \tau\beta_1\eta_1 + \theta_1 + \sqrt{4\beta_1(s_p^3\beta_1 - s\tau\beta_1)\eta_1 + (-s_p^3\beta_1 + \beta_1\eta_1 - \tau\beta_1\eta_1 - \theta_1 - \mu_1)^2} + \mu_1}{2\beta_1\eta_1}$$

Substituting s_{p1}^3 in w_{p1}^3 and solving for w_{p1}^3 we obtain four solutions $w_{p11}^3 = 0$, two real roots $(w_{p12}^3 and\ w_{p13}^3)$ and a complex root w_{p14}^3 (See the attached Mathematica file 1). Note that $w_{p11}^3 = 0$ correspond to $\mathbf{E^2}$. Complex root is not feasible in this case and hence discarded

Substituting s_{p1}^3 in w_{p2}^3 and solving for w_{p2}^3 we obtain four solutions $w_{p21}^3 = 0$, two real roots $(w_{p22}^3 and\ w_{p23}^3)$ and a complex root w_{p24}^3 (See the attached Mathematica file 2). Note that $w_{p11}^3 = 0$ correspond to $\mathbf{E^2}$. Complex root is not feasible in this case and hence discarded

Substituting s_{p2}^3 in w_{p1}^3 and solving for w_{p1}^3 we obtain four solutions $w_{p111}^3 = 0$, two real roots $(w_{p112}^3 and\ w_{p113}^3)$ and a complex root w_{p114}^3 (See the attached Mathematica file 3). Note that $w_{p111}^3 = 0$ correspond to $\mathbf{E^2}$. Complex root is not feasible in this case and hence discarded

Substituting s_{p2}^3 in w_{p2}^3 and solving for w_{p2}^3 we obtain four solutions $w_{p121}^3 = 0$, two real roots $(w_{p122}^3 and\ w_{p123}^3)$ and a complex root w_{p124}^3 (See the attached Mathematica file 4). Note that $w_{p121}^3 = 0$ correspond to $\mathbf{E^2}$. Complex root is not feasible in this case and hence discarded.

Analytical solution point of the equation point to the possibility of at most eight endemic equilibrium point however numerical solution will be used to confirm the feasible one later in the paper.

3.3.2 Stability of Steady states

3.3.2.1 Local stability of steady states

Generally, local stability of the steady state is determined to linearize the systems of equations (3.2.1) and (3.2.1) to obtain a jacobian matrix (J) below.

$$\begin{pmatrix} (\tau - s_p)\beta_2 - \beta_2(s_p + w_p\eta_2) - \theta_2 - \mu_2 & (\tau - s_p)\beta_2\eta_2 \\ (1 - \tau - w_p)\beta_1 & (1 - \tau - w_p)\beta_1\eta_1 - \beta_1(s_p + w_p\eta_1) - \theta_1 - \mu_1 \end{pmatrix}$$

Trace

$$((\tau - s_p)\beta_2 - \beta_2(s_p + w_p\eta_2) - \theta_2 - \mu_2) + (1 - \tau - w_p)\beta_1\eta_1 - \beta_1(s_p + w_p\eta_1) - \theta_1 - \mu_1 < 0$$

$$\tau\beta_2 + \beta_1\eta_1 < s_p(\beta_1 + 2\beta_2) + (\tau + 2w_p)\beta_1\eta_1 + w_p\beta_2\eta_2 + \theta_1 + \theta_2 + \mu_1 + \mu_2$$

Determinant

$$\{((\tau - s_p)\beta_2 - \beta_2(s_p + w_p\eta_2) - \theta_2 - \mu_2)((1 - \tau - w_p)\beta_1\eta_1 - \beta_1(s_p + w_p\eta_1) - \theta_1 - \mu_1)\}$$
$$- \{((1 - \tau - w_p)\beta_1)((\tau - s_p)\beta_2\eta_2)\} > 0$$
$$2s_p^2\beta_1\beta_2 + (\theta_1 + \mu_1)(w_p\beta_2\eta_2 + \theta_2 + \mu_2) + \beta_1(\beta_2(\tau\eta_1 + (\tau^2 + w_p(\tau + (\tau + 2w_p)\eta_1))\eta_2) + (\tau$$
$$+ 2w_p)\eta_1(\theta_2 + \mu_2)) + s_p(2\beta_2(\theta_1 + \mu_1) + \beta_1(\beta_2(2(\tau + 2w_p)\eta_1 + \eta_2) + \theta_2 + \mu_2))$$
$$> s_p\beta_1\beta_2(\tau + 2\eta_1 + \tau\eta_2) + \tau\beta_2(\theta_1 + \mu_1) + \beta_1(\beta_2(\tau\eta_2 + \eta_1(\tau^2 + w_p(2\tau + \eta_2))))$$
$$+ \eta_1(\theta_2 + \mu_2))$$

The specific conditions necessary for each of the steady states are obtained by substituting s_p and w_p in determinant and trace by their corresponding expressions or values at each of the steady states($E^0, E^1, E^2,$ and E^3).

3.3.2.2 Global stability of steady states

Let s_p^* *and* w_p^* be the general steady states of the system of equations (3.2.1) and (3.2.1), we propose the following Lyapunov function

$$V(s_p, w_p) = \int_{s_p^*}^{s_p} \frac{\xi - s_p^*}{\xi} d\xi + q_1 \int_{w_p^*}^{w_p} \frac{\omega - w_p^*}{\omega} d\omega$$

$$\dot{V} = \frac{dV}{dt} = \frac{s_p - s_p^*}{s_p}\frac{ds_p}{dt} + q_1\frac{w_p - w_p^*}{w_p}\frac{dw_p}{dt}$$

$$\dot{V} = \frac{dV}{dt} = \frac{s_p - s_p^*}{s_p}\left(\beta_2(s_p + \eta_2 w_p)(\tau - s_p) - (\theta_2 + \mu_2)s_p\right)$$
$$+ q_1\frac{w_p - w_p^*}{w_p}\left(\beta_1(s_p + \eta_1 w_p)(1 - \tau - w_p) - (\theta_1 + \mu_1)w_p\right)$$

$$\dot{V} = \frac{dV}{dt} = \frac{s_p - s_p^*}{s_p}\left(\tau s_p\beta_2 + \tau w_p\beta_2\eta_2 - s_p^2\beta_2 - s_p w_p\beta_2\eta_2 - s_p\theta_2 - s_p\mu_2\right)$$
$$+ q_1\frac{w_p - w_p^*}{w_p}\left(s_p\beta_1 + w_p\beta_1\eta_1 - \tau s_p\beta_1 - s_p w_p\beta_1 - \tau w_p\beta_1\eta_1 - w_p^2\beta_1\eta_1 - w_p\theta_1\right.$$
$$\left. - w_p\mu_1\right)$$

$$\dot{V} = \frac{dV}{dt} = (\tau s_p\beta_2 + \tau w_p\beta_2\eta_2) - (s_p^2\beta_2 + s_p w_p\beta_2\eta_2 + s_p\theta_2 + s_p\mu_2) - \frac{s_p^*}{s_p}(\tau s_p\beta_2 + \tau w_p\beta_2\eta_2) + \frac{s_p^*}{s_p}(s_p^2\beta_2$$
$$+ s_p w_p\beta_2\eta_2 + s_p\theta_2 + s_p\mu_2)$$
$$+ q_1(s_p\beta_1 + w_p\beta_1\eta_1) - q_1(\tau s_p\beta_1 + s_p w_p\beta_1 + \tau w_p\beta_1\eta_1 + w_p^2\beta_1\eta_1 + w_p\theta_1 - w_p\mu_1)$$
$$- q_1\frac{w_p^*}{w_p}s_p\beta_1 + w_p\beta_1\eta_1) + q_1\frac{w_p^*}{w_p}(\tau s_p\beta_1 + s_p w_p\beta_1 + \tau w_p\beta_1\eta_1 + w_p^2\beta_1\eta_1 + w_p\theta_1$$
$$+ w_p\mu_1),$$

Set w_p to zero

$$\tau w_p\beta_2\eta_2 + q_1 w_p\beta_1\eta_1 - q_1(\tau w_p\beta_1\eta_1 + w_p\theta_1 - w_p\mu_1) = 0$$

$$\tau\beta_2\eta_2 + q_1\beta_1\eta_1 - q_1(\tau\beta_1\eta_1 + \theta_1 - \mu_1) = 0$$

$$q_1 = \frac{\tau\beta_2\eta_2 + q_1\beta_1\eta_1}{(\tau\beta_1\eta_1 + \theta_1 - \mu_1)}$$

$$\dot{V} = \frac{dV}{dt} = \tau s_p\beta_2 - (s_p^2\beta_2 + s_pw_p\beta_2\eta_2 + s_p\theta_2 + s_p\mu_2) - \frac{s_p^*}{s_p}(\tau s_p\beta_2 + \tau w_p\beta_2\eta_2) + \frac{s_p^*}{s_p}(s_p^2\beta_2 + s_pw_p\beta_2\eta_2$$
$$+ s_p\theta_2 + s_p\mu_2) + \frac{\tau\beta_2\eta_2 + q_1\beta_1\eta_1}{(\tau\beta_1\eta_1 + \theta_1 - \mu_1)}s_p\beta_1 - \frac{\tau\beta_2\eta_2 + q_1\beta_1\eta_1}{(\tau\beta_1\eta_1 + \theta_1 - \mu_1)}(\tau s_p\beta_1 + s_pw_p\beta_1$$
$$+ w_p^2\beta_1\eta_1) - \frac{\tau\beta_2\eta_2 + q_1\beta_1\eta_1}{(\tau\beta_1\eta_1 + \theta_1 - \mu_1)}\frac{w_p^*}{w_p}(s_p\beta_1 + w_p\beta_1\eta_1) + \frac{\tau\beta_2\eta_2 + q_1\beta_1\eta_1}{(\tau\beta_1\eta_1 + \theta_1 - \mu_1)}\frac{w_p^*}{w_p}(\tau s_p\beta_1$$
$$+ s_pw_p\beta_1 + \tau w_p\beta_1\eta_1 + w_p^2\beta_1\eta_1 + w_p\theta_1 + w_p\mu_1),$$

Let

$$P = \tau s_p\beta_2 + \frac{s_p^*}{s_p}(s_p^2\beta_2 + s_pw_p\beta_2\eta_2 + s_p\theta_2 + s_p\mu_2) + \frac{\tau\beta_2\eta_2 + q_1\beta_1\eta_1}{(\tau\beta_1\eta_1 + \theta_1 - \mu_1)}s_p\beta_1$$
$$+ \frac{\tau\beta_2\eta_2 + q_1\beta_1\eta_1}{(\tau\beta_1\eta_1 + \theta_1 - \mu_1)}\frac{w_p^*}{w_p}(\tau s_p\beta_1 + s_pw_p\beta_1 + \tau w_p\beta_1\eta_1 + w_p^2\beta_1\eta_1 + w_p\theta_1 + w_p\mu_1),$$

$$Q = (s_p^2\beta_2 + s_pw_p\beta_2\eta_2 + s_p\theta_2 + s_p\mu_2) + \frac{s_p^*}{s_p}(\tau s_p\beta_2 + \tau w_p\beta_2\eta_2)$$
$$+ \frac{\tau\beta_2\eta_2 + q_1\beta_1\eta_1}{(\tau\beta_1\eta_1 + \theta_1 - \mu_1)}(\tau s_p\beta_1 + s_pw_p\beta_1 + w_p^2\beta_1\eta_1) + \frac{\tau\beta_2\eta_2 + q_1\beta_1\eta_1}{(\tau\beta_1\eta_1 + \theta_1 - \mu_1)}\frac{w_p^*}{w_p}(s_p\beta_1$$
$$+ w_p\beta_1\eta_1)$$

The steady states generally expressed as $E^* = \{s_p^*, w_p^*\}$ are globally asymptotically

Stable whenever $Q > P$. The conditions necessary for each of the steady states are obtained by substituting each of the steady states.

4 Parametrization and Results

Parametrization of a model using real data is a critical part of model validation however availability of secondary data always poses a major challenge. The aim of this paper was model formulation and analysis with the hope of fitting the model in real data in a subsequent paper.

We expect:

i. $\beta_1 > \beta_2$ Because the student is more likely to be corrupted in accessing service.
ii. $\tau \gg 0.5$ Because the proportion of student is always much greater than staff in any institution
iii. In Kenya, most of the degree programmes last four years hence $\mu_1 \cong \frac{1}{4}year^{-1}$ however on average staff serve in Higher institutions much longer. Let us assume each academic staff serve in an institution for an average of 8 years then $\mu_2 = \frac{1}{8}year^{-1}$
iv. $\theta_1 > \theta_2$ Voluntary quitting of corrupt practices is expected to be more in students than in staff because students are motivated by short term goals.
v. $\eta_2 > 0.5$ and $\eta_1 > 0.5$ We expect students to be drivers of corruption in institutions however students are more likely to influence one another more to corrupt practices than to staff hence $\eta_1 \geq \eta_2$

Consider a hypothetical institution with a student population of 4000 and a staff population of 100. Suppose 5 percent of students are corrupt and 1 percent of staff are corrupt. Again let us assume 2% of corrupt students and 1% of corrupt staff voluntarily quit corruption every year.

$$s_p(0) = \frac{\frac{1}{100} \times 100}{4000 + 100} = \frac{1}{4100}; \; w_p(0) = \frac{\frac{5}{100} \times 4000}{4000 + 100}; \; \theta_2 = \frac{\frac{1}{100} \times 1}{1} = \frac{1}{100}; \; \theta_1 = \frac{\frac{2}{100} \times 200}{200};$$

$$\tau = \frac{4000}{4100}$$

Let $\beta_1 = 5$; $\beta_2 = 2$; $\eta_2 = 0.55$; $\eta_1 = 0.6$

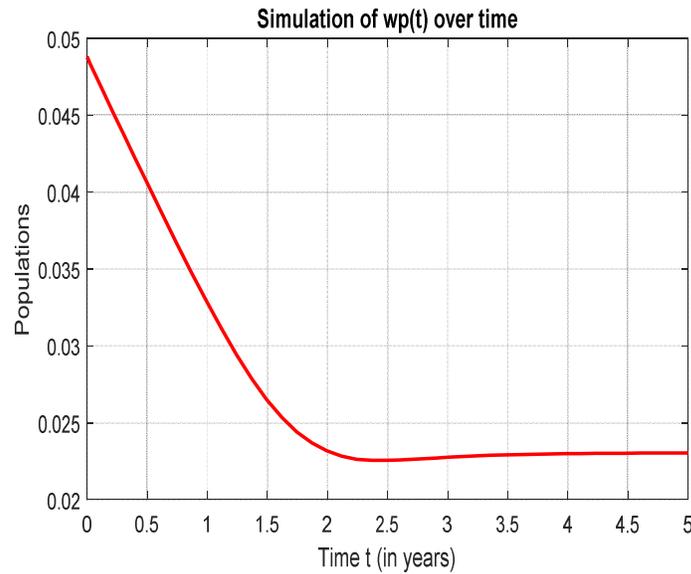Based on the above parameters and data, the numerical simulations of $s_p(t) \; and \; w_p(t)$ obtained using Matlab Ode solvers for five years was as follows

Fig. 1. Simulation of Wp(t) with time
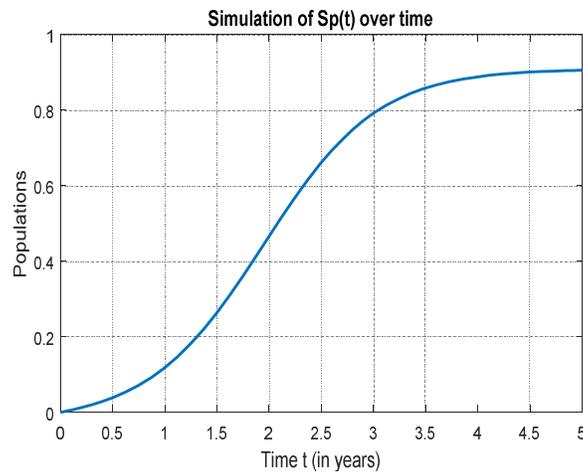
Fig. 2. Simulation of Sp(t) with time

Fig. 1 and Fig. 2 describes how $w_p(t) \; and \; s_p(t)$ would change over five years
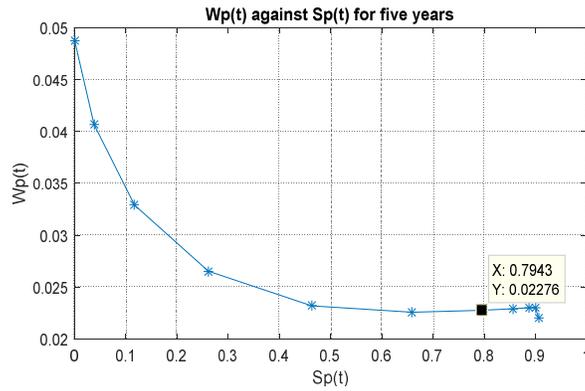
Fig. 3. Wp(t) against Sp(t) over five years

Fig. 3. Describes how $w_p(t)$ *changes* $s_p(t)$ over five years

The numerical solution of $E^3 = \{s_p^3, w_p^3\} = \{0.41928, 0.01531\}$ which confirms the numerical simulations.

5 Numerical Bifurcation Analysis
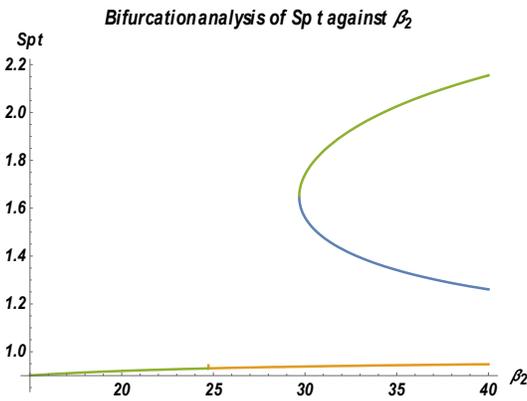
Let β_1 be the bifurcation parameter

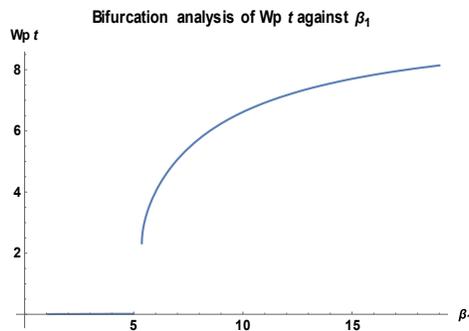**Fig. 4. Bifurcation analysis of Sp(t) against beta2**

Fig. 5. Bifurcation analysis of Sp(t) against Beta 1

Figs. 4 and 5 show bifurcation analysis of s_p( )   ( )(t) and wp(t) with respect to β_2 and β_1 respectively.
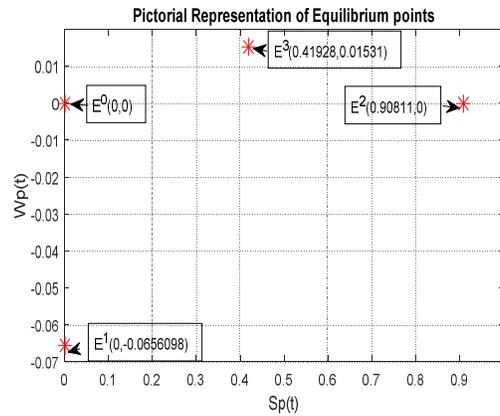

Fig. 6. Pictorial representation of equilibrium points

Fig. 6. Show the numerical values of the stead states as follows:

$E^0 = \{0,0\}, E^1 = \{0, -0.0656\}, E^2 = \{0.90811,0\}$ and $E^3 = \{0.41928,0.01531\}$

6 Results and Discussion

Based on the theoretical parameter, the numerical results in Fig. 6 indicate that the system has three feasible equilibrium points E^0, E^2 and E^3 because E^1 has negative value.

Fig. 4 indicates backward bifurcation exist when β_2 is about 39 suggesting a possibility of two equilibria.

Figs. 1,2,3 show that the corrupt student would decrease with time while corrupt staff would increase with time because staff stay longer in an institution than students. As a result any corruption mitigation strategy should focus more on staff than student.

7 Conclusion

Numerical bifurcation analysis suggests the possibility of backward of corrupt staff when β_2 is about 39. Numerical simulation points to an increasing trend on corrupt staff and decrease trend on corrupt student. This study concludes that more focus should be put to staff than students in curbing the spread of corruption due to their longevity of stay in work station.

8 Recommendation

 Future studies should strive to include all components of Higher institution system like sub ordinate staff, suppliers among others. Fitting this model to real data to parametrize the model should also be a priority.

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Competing Interests

Authors have declared that no competing interests exist.

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